

فصل چهارم جبر و احتمال

احتمال هم شانس در فضاهای گسته

هزار	۱ راه اول : $s = \{1, 2, 3, \dots, 10\} \quad n(s) = 10 \quad (\cdot / 25) \quad A = \{1, 3, 5, 7, 9\} \quad (\cdot / 25) \quad B = \{1, 2, 3, 4, 5\} \quad (\cdot / 25)$ $A \cup B = \{1, 2, 3, 4, 5, 7, 9\} \quad (\cdot / 25) \quad n(A \cup B) = 7 \quad (\cdot / 25) \quad P(A \cup B) = \frac{n(A \cup B)}{n(s)} = \frac{7}{10} \quad (\cdot / 5)$ راه دوم : $A = \{1, 3, 5, 7, 9\} \quad P(A) = \frac{5}{10} \quad (\cdot / 25) \quad B = \{1, 2, 3, 4, 5\} \quad P(B) = \frac{5}{10} \quad (\cdot / 25)$ $A \cap B = \{1, 3, 5\} \quad (\cdot / 25) \quad P(A \cap B) = \frac{3}{10} \quad (\cdot / 25) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (\cdot / 25)$ $P(A \cup B) = \frac{5}{10} + \frac{5}{10} - \frac{3}{10} = \frac{7}{10} \quad (\cdot / 5)$
هزار	۲ $n(s) = \binom{32}{2} \quad (\cdot / 25)$ الف) $P(A) = \frac{\binom{8}{2}}{\binom{32}{2}} \quad (\cdot / 25)$ ب) $P(B) = \frac{\binom{8}{1}\binom{8}{1}}{\binom{32}{2}} \quad (\cdot / 5)$
هزار	۳ $n(s) = \binom{12}{4} \quad (\cdot / 25)$ $n(A) = \binom{4}{1}\binom{8}{1} + \binom{4}{2}\binom{8}{2} \quad (\cdot / 5)$ $P(A) = \frac{n(A)}{n(s)} = \frac{\binom{4}{1}\binom{8}{1} + \binom{4}{2}\binom{8}{2}}{\binom{12}{4}} \quad (\cdot / 5)$
هزار	۴ الف) $P(A) = \frac{\binom{5}{3}\binom{6}{2}}{\binom{11}{5}} = \frac{25}{77} \quad (\cdot / 25)$ ب) $P(B) = \frac{\binom{5}{5}\binom{6}{1}}{\binom{11}{5}} = \frac{1}{462} \quad (\cdot / 5)$

٤	<p>خ د د ٦</p> <p>الف) $p(A) = \frac{\binom{5}{1} \binom{5}{1}}{\binom{10}{2}} = \frac{5 \times 5}{45} = \frac{1}{9}$ (✓/٧٥)</p> <p>ب) $p(B) = \frac{\binom{4}{1} \binom{5}{1} + \binom{4}{2} \binom{5}{0}}{\binom{10}{2}} = \frac{4 \times 5 + 1 \times 1}{45} = \frac{21}{45} = \frac{1}{5}$ (١)</p>
٥	<p>ش ب ر ٨٤</p> <p>الف) $p(A) = \frac{\binom{7}{1} \binom{5}{3}}{\binom{12}{4}} = \frac{7 \times 10}{495} = \frac{14}{495}$ (✓/٧٥)</p> <p>ب) $p(B) = \frac{\binom{7}{3} \binom{5}{1} + \binom{7}{4} \binom{5}{0}}{\binom{12}{4}} = \frac{35 \times 5 + 35}{495} = \frac{210}{495}$ (١)</p>
٦	<p>٨٤</p> <p>الف) $P(A) = \frac{\binom{5}{2}}{\binom{8}{2}} = \frac{10}{28} = \frac{5}{14}$ (✓/٧٥)</p> <p>ب) $P(B) = \frac{\binom{5}{1} \binom{3}{1}}{\binom{8}{2}} = \frac{5 \times 3}{28} = \frac{15}{28}$ (✓/٧٥)</p>
٧	<p>خ د د ٨٧</p> <p>$p(A) = \frac{7 \times 6 \times 5 \times 4 \times 3}{7^5} = \frac{360}{2401}$ (١)</p>
٩	<p>خ د د ٨٧</p> <p>$P(A) = \frac{\binom{5}{1} \binom{7}{2} + \binom{5}{2} \binom{7}{1}}{\binom{12}{4}} = \frac{5 \times 21 + 10 \times 7}{220} = \frac{175}{220} = \frac{35}{44}$ (✓/٥)</p>

١٠	$P(A) = \frac{\binom{5}{2} \binom{7}{2}}{\binom{12}{4}} = \frac{10 \times 21}{495} = \frac{14}{33}$ (٠/٥)	
١١	$n(S) = \binom{10}{5} = 252$ (٠/٥) الف) $n(A) = \binom{6}{2} \times \binom{4}{2} = 120$ (٠/٢٥) $\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{120}{252} = \frac{10}{21}$ (٠/٢٥) ب) $n(B) = \binom{6}{5} = 6$ (٠/٢٥) $\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{6}{252} = \frac{1}{42}$ (٠/٢٥)	١١
١٢	$n(s) = \binom{16}{3} = 560$ (٠/٢٥) ب) $P(B) = \frac{\binom{10}{2} \binom{6}{1}}{\binom{16}{3}} = \frac{27}{56}$ (٠/٧٥) ، الف) $P(A) = \frac{n(A)}{n(s)} = \frac{\binom{6}{2}}{\binom{16}{3}} = \frac{1}{28}$ (٠/٥)	١٢
١٣	$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 3, 9, 15, 21, 27\}$ $\rightarrow P(A) = \frac{20}{30} = \frac{2}{3}$ (٠/٧٥) ب) $B = \{6, 12, 18, 24, 30\} \rightarrow P(B) = \frac{5}{30} = \frac{1}{6}$ (٠/٧٥)	١٣
١٤	الف) $P(A) = \frac{C(5, 2)}{C(11, 2)} = \frac{10}{55} = \frac{2}{11}$ (٠/٢٥) ب) $P(B) = \frac{C(5, 2)}{C(11, 2)} + \frac{C(6, 2)}{C(11, 2)} = \frac{2}{11} + \frac{15}{55} = \frac{2}{11} + \frac{3}{11} = \frac{5}{11}$ (٠/٢٥)	١٤

۱۵

خرداد ۹۸

$$\text{الف) } p(A) = \frac{\binom{6}{2} \binom{4}{1} + \binom{6}{3}}{\binom{10}{3}} \quad (\cdot / ۲۵)$$

$$= \frac{15 \times 4 + 20}{120} = \frac{۲}{۳} \quad (\cdot / ۲۵)$$

$$\text{ب) } p(B) = \frac{\binom{4}{1}}{\binom{10}{3}} = \frac{4}{120} = \frac{1}{30} \quad (\cdot / ۲۵)$$

$$(\cdot / ۵) \quad (\cdot / ۲۵)$$

۱۶

شهریور ۸۹

هر قسمت نوشته شده صورت کسر $(\cdot / ۲۵)$

$$P(A) = \frac{\binom{1}{1} \binom{2}{1} \binom{3}{1} \binom{4}{1}}{\binom{10}{4}} = \frac{۲۴}{210} = \frac{4}{35}$$

$$(\cdot / ۲۵)$$

۱۷

۸۹

$$P(A) = \frac{7 \times 6 \times 5 \times 4}{7!} \quad (\cdot / ۵)$$

صورت کسر هر قسمت $(\cdot / ۲۵)$ و مخرج کسر $(\cdot / ۵)$

۱۸

خرداد ۹۰

$$p(A) = \frac{\binom{4}{2} \binom{6}{1} + \binom{4}{3}}{\binom{10}{3}} = \frac{۴۰}{120} = \frac{1}{3} \quad (\cdot / ۲۵)$$

$$(\cdot / ۲۵)$$

۲۰

شهریور ۹۰

$$P(A) = \frac{\binom{5}{1} \binom{10}{2} + \binom{5}{2} \binom{10}{1} + \binom{5}{3}}{\binom{15}{3}} = \frac{۴۰}{۵۶} = \frac{11}{14} \quad (\cdot / ۲۵)$$

صورت کسر هر قسمت نوشته شده $\cdot / ۲۵$ نمره و مخرج کسر $\cdot / ۵$ نمره داده شود.

٢٢

$$n(S) = ٤^٣ = ٦ \quad (\cdot / ٢٥)$$

٩٠

$$\text{الف) } n(A) = \binom{٣}{٢} + \binom{٣}{١} = ٣ + ١ = ٤ \quad (\cdot / ٥)$$

$$p(A) = \frac{n(A)}{n(S)} = \frac{٤}{٦} = \frac{٢}{٣} \quad (\cdot / ٢٥)$$

$$\text{ب) } n(B) = \binom{٣}{٢} + \binom{٣}{٠} = ٣ + ١ = ٤ \quad (\cdot / ٢٥) \quad p(B) = \frac{n(B)}{n(S)} = \frac{٤}{٦} = \frac{٢}{٣} \quad (\cdot / ٢٥)$$

٢٣

٩١

$$\text{الف) } p(A) = \frac{\binom{٥}{٢} \times \binom{١٠}{١}}{\binom{١٥}{٣}} = \frac{٣٠}{٤٥٥} = \frac{٦}{٩١} \quad (\cdot / ٢٥)$$

$$\text{ب) } p(B) = \frac{\binom{٥}{١} \times \binom{٥}{٢} \times \binom{٤}{١}}{\binom{١٥}{٣}} = \frac{١٢٠}{٤٥٥} = \frac{٢٤}{٩١} \quad (\cdot / ٢٥)$$

٢٤

٩٢

$$n(S) = \binom{١١}{٣} = ١٦٥ \quad (\cdot / ٢٥) \quad n(A) = \binom{٥}{٢} \binom{٦}{١} + \binom{٥}{٣} = ٧٠ \quad (\cdot / ٧٥)$$

$$p(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) \quad \Rightarrow p(A) = \frac{٧٠}{١٦٥} = \frac{٢}{٣} \quad (\cdot / ٢٥)$$

٢٥

٩٣

$$S = \{١R, ٢R, ٣R, ٤R, ٥R, ٦R, PRR, PRP, PPR, PPP\} \Rightarrow n(s) = ١٥ \quad (\cdot / ٥)$$

$$A = \{٢R, ٤R, ٦R\} \Rightarrow n(A) = ٣ \Rightarrow p(A) = \frac{٣}{١٥} \quad (\cdot / ٧٥)$$

$$B = \{PPR, PRP\} \Rightarrow n(B) = ٢ \Rightarrow P(B) = \frac{٢}{١٥} = \frac{١}{٥} \quad (\cdot / ٧٥)$$

٢٦

٩٤

$$S = \{ddd, ddp, dpd, pdd, ppp, ppd, pdp, dpp\} \Rightarrow n(s) = ٨ \quad (\cdot / ٨)$$

$$A = \{ppp, ppd, pdp, dpp\} \Rightarrow n(A) = ٤ \Rightarrow p(A) = \frac{٤}{٨} \quad (\cdot / ٨)$$

$$B = \{ddd, ddp, dpd, dpp\} \Rightarrow n(B) = ٤ \Rightarrow P(B) = \frac{٤}{٨} \quad (\cdot / ٨)$$

٩٢ $n(S) = \binom{10}{2} = 45 \quad (\cdot / ٢٥)$ $n(A) = \binom{5}{1} \times \binom{5}{1} = 25 \quad (\cdot / ٢٥)$ $P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) \Rightarrow P(A) = \frac{25}{45} = \frac{1}{2} \quad (\cdot / ٢٥)$	٢٨
٩٣ $p(A) = \frac{C(\text{ف}, \text{ر}) + C(\text{ر}, \text{ف})}{C(\text{ل}, \text{ر})} = \frac{9}{18} \quad (\cdot / ٢٥)$	٢٩
٩٤ $P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) = \frac{365 \times 364 \times \dots \times (365-1+1)}{365^{10}} \quad (\cdot / ٢٥)$	٣٠
٩٥ $n(s) = \binom{12}{2} \quad (\cdot / ٢٥)$ $\text{ا) } P(A) = \frac{\binom{6}{2} \times \binom{6}{1}}{\binom{12}{2}} \quad (\cdot / ٥) = \frac{45 \times (12/2)}{66} = \frac{45}{66}$ $\text{ب) } P(B) = \frac{\binom{6}{1}}{\binom{12}{2}} \quad (\cdot / ٢٥) = \frac{12/2}{66} = \frac{1}{66} \quad (\cdot / ٢٥)$	٣١
٩٦ $n(s) = 2 \times 4 \times 3 \times 2 = 48 \quad (\cdot / ٥)$ $n(A) = 2 \times 3 \times 2 \times 1 = 12 \quad (\cdot / ٥)$ $P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) = \frac{12}{48} = \frac{1}{4} \quad (\cdot / ٢٥)$	٣٢
٩٧ $n(s) = \binom{12}{1} = 66 \quad (\cdot / ٥)$ $n(A) = \binom{6}{1} \times \binom{6}{1} = 36 \quad (\cdot / ٢٥)$ $P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) = \frac{36}{66} \quad (\cdot / ٢٥)$	٣٣

٣٥	<p>الف) $n(S) = \binom{9}{3} = \frac{9!}{3! \times 6!} = 84 \quad (\cdot / ٢٥)$</p> $P(A) = \frac{n(A)}{n(S)} = \frac{\binom{4}{2}}{84} \quad (\cdot / ٢٥) = \frac{6}{84} = \frac{1}{14}$ <p>ب) $n(B) = \binom{9}{2} \times \binom{5}{1} \quad (\cdot / ٢٥) + \binom{4}{3} \binom{5}{1} \quad (\cdot / ٢٥) = ٣٤ \quad p(B) = \frac{34}{84} \quad (\cdot / ٢٥) = \frac{١٧}{٤٢}$</p>
٣٦	$n(S) = \binom{7}{2} \quad (\cdot / ٢٥) \quad , \quad n(A) = \binom{4}{0} \binom{3}{2} + \binom{4}{1} \binom{3}{1} \quad (١) \quad \Rightarrow \quad P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) = \frac{15}{21}$
٣٧	$n(S) = ٥ \times ٤ \times ٣ = ٦٠ \quad (\cdot / ٢٥) , \quad n(A) = ٤ \times ٣ \times ٢ = ٢٤ \quad (\cdot / ٥) \Rightarrow \quad P(A) = \frac{n(A)}{n(S)} \quad (\cdot / ٢٥) = \frac{٢}{٥}$
	احتمال دو جمله ای
٤١	$P(A) = \frac{٣}{٦} = \frac{١}{٢} \quad (\cdot / ٢٥)$ <p>احتمال زوج آمدن یک بار پرتاب تاس</p>
٤٢	$P(B) = \frac{\binom{٥}{٢}}{٢٥} = \frac{١٠}{٣٢} = \frac{٥}{١٦} \quad (\cdot / ٢٥)$
٤٣	$P(A) = \frac{\binom{٦}{٢}}{٢٦} = \frac{١٥}{٦٤} \quad (\cdot / ٢٥)$
٤٤	$p(A) = \frac{\binom{١٥}{٣}}{٢١٥} \quad (\cdot / ٧٥)$

هزار	$P(A) = \frac{\binom{8}{2}}{\binom{12}{5}} + \frac{\binom{8}{3}}{\binom{12}{5}} + \frac{\binom{8}{4}}{\binom{12}{5}} = \frac{28}{256} + \frac{8}{256} + \frac{1}{256} = \frac{37}{256} \quad (+/25)$	✓
شانزده	$p(A) = \frac{\binom{4}{3} \cdot \binom{5}{2}}{\binom{12}{5}} = \frac{20}{128}$	✓
نیم	جای خالی به ترتیب برابر است با: $\frac{1}{4}, \frac{2}{4} = \frac{1}{2}, \frac{1}{4}$ رسم نمودار درختی $(+/25)$ هر قسمت جواب $(+/25)$	✓
هزار	$p(A) = \frac{\binom{10}{3} \cdot \binom{5}{2}}{\binom{12}{5}}$	✓
شانزده	$P(A) = \frac{\binom{10}{2} \cdot \binom{5}{3}}{\binom{12}{5}}$	✓
نیم	$P(A) = \frac{\binom{5}{2}}{\binom{5}{5}}$ صورت کسر $(+/5)$ و مخرج کسر $(+/5)$	✓

٩٠	$p(A) = \frac{\binom{20}{5} (0.2)^5 (0.8)^{15}}{\binom{20}{0} (0.2)^0 (0.8)^{20}}$	١٠
٩١	$P = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{12}{4}}{\binom{12}{12}}$ صورت و مخرج کسرهای قسمت (٠/٥)	١١
٩٢	$p(A) = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{12}{4}}{\binom{12}{12}}$	١٢
٩٣	$p(A) = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{12}{4}}{\binom{12}{12}}$	١٣
٩٤	$p(A) = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{12}{4}}{\binom{12}{12}}$	١٤
٩٥	$p(A) = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{12}{4}}{\binom{12}{12}}$	١٥
٩٦	$P = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{10}{4}}{\binom{10}{10}}$ صورت و مخرج کسرهای قسمت (٠/٥)	١٦
٩٧	$P = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{10}{4}}{\binom{10}{10}}$ صورت و مخرج کسرهای قسمت (٠/٥)	١٧
٩٨	$p(A) = \frac{\binom{5}{3}}{\binom{5}{5}} = \frac{10}{32} = \frac{5}{16}$ (٠/٢٥)	١٨
٩٩	$P = \frac{\binom{n}{k}}{\binom{n}{n}} = \frac{\binom{10}{4}}{\binom{10}{10}}$ صورت و مخرج کسرهای قسمت (٠/٥)	١٩

خرداد ۹۳	$P(A) = \frac{\binom{n}{k}}{2^n} = \frac{\binom{k}{\alpha} + \binom{k}{\beta} + \binom{k}{\gamma}}{2^k}$ (۰/۲۵)	۲۰
خرداد ۹۴	<p style="text-align: center;">رسم نمودار درختی (۰/۰)</p> <p style="text-align: right;">تعداد پسرها: ۱ ۲ تعداد حالات: $\frac{1}{4}$ $\frac{1}{2}$ احتمال: ۹۴ ص</p>	۲۱
شهریور ۹۴	$P(\text{بازار رو بیاید}) = \frac{\binom{15}{7}}{2^{15}} \quad (۰/۰)$ (۰/۰)	۲۲
	احتمال غیر هم شانس در فضاهای گسته و احتمال یک پیشامد اختیاری	
خرداد ۸۵	$P(A) = P(C) = \tau P(B) \quad (۰/۰)$ $P(A) + P(B) + P(C) = ۱ \quad (۰/۰) \Rightarrow \tau P(B) + P(B) + \tau P(B) = ۱ \quad (۰/۰)$ $\tau P(B) = ۱ \Rightarrow P(B) = \frac{۱}{\tau} \quad (۰/۰) \quad P(A) = P(C) = \tau \left(\frac{۱}{\tau} \right) = \frac{۱}{\tau} \quad (۰/۰)$ $P(B \cup C) = P(B) + P(C) = \frac{۱}{\tau} + \frac{۱}{\tau} = \frac{۲}{\tau} \quad (۰/۰)$	۱
شهریور ۸۵	$P(a) + P(b) + P(c) + P(d) = ۱ \quad (۰/۰)$ $\tau P(b) + P(b) + \frac{۱}{\tau} + \frac{۱}{\tau} = ۱ \quad (۰/۰)$ $\tau P(b) + \frac{۱}{\tau} = ۱$ $\tau P(b) = \frac{۱}{\tau} \quad P(b) = \frac{۱}{\tau} \quad (۰/۰)$ $P(a) = \tau P(b) = \tau \left(\frac{۱}{\tau} \right) = \frac{۱}{\tau} \quad (۰/۰)$ $P(a') = ۱ - P(a) = ۱ - \frac{۱}{\tau} = \frac{۲}{\tau} \quad (۰/۰)$ $P(b') = ۱ - P(b) = ۱ - \frac{۱}{\tau} = \frac{۰}{\tau} \quad (۰/۰)$	۲

ج ذ ك	$P(\text{ا}) = P(\text{ب}) = P(\text{ج}) = \omega$ $P(\text{د}) = P(\text{ه}) = P(\text{ز}) = \text{ر}\omega \quad (\cdot / ٣\Delta)$ $P(\text{ا}) + P(\text{ب}) + P(\text{ج}) + P(\text{ه}) + P(\text{ز}) + P(\text{ز}) = ١ \quad (\cdot / ٣\Delta)$ $\omega + \text{ر}\omega + \omega + \text{ر}\omega + \omega + \text{ر}\omega = ١ \Rightarrow ٦\omega = ١ \Rightarrow \omega = \frac{١}{٦} \quad (\cdot / ٣\Delta) \quad P(\text{ا}) = P(\text{ب}) = P(\text{ج}) = \frac{١}{٦} \quad (\cdot / ٣\Delta)$ $P(\text{د}) = P(\text{ه}) = P(\text{ز}) = \text{ر}\left(\frac{١}{٦}\right) = \frac{\text{ر}}{٦} \quad (\cdot / ٣\Delta) \quad P(A) = P(\text{ا}) + P(\text{ب}) + P(\text{ج}) = \frac{١}{٦} + \frac{\text{ر}}{٦} + \frac{١}{٦} = \frac{\omega}{٦} \quad (\cdot / ٣\Delta)$	ج
ج ذ ك	$p\{a, b, c\} + p\{d\} = ١ \quad (\cdot / ٤\Delta)$ $p\{d\} = ١ - p\{a, b, c\} = ١ - \frac{٣}{٤} = \frac{١}{٤} \quad (\cdot / \Delta)$ $p\{a, b\} = \frac{٣}{٤} - p\{d\} = \frac{٣}{٤} - \frac{١}{٤} = \frac{٢}{٤} = \frac{١}{٢} \quad (\cdot / \Delta)$	ج
ج ذ ك	$p(A_1) + p(A_{\text{ر}}) + p(A_{\text{ز}}) + p(A_{\text{ه}}) = ١ \quad (\cdot / ٤\Delta)$ $p(A_{\text{ر}}) = \text{ر}p(A_{\text{ه}})$ $p(A_1) = \text{ر}p(A_{\text{ر}}) = \text{ر}^2p(A_{\text{ه}}) \quad (\cdot / \Delta)$ $p(A_{\text{ز}}) = \text{ر}p(A_{\text{ه}})$ $\text{ر}p(A_{\text{ه}}) + \text{ر}p(A_{\text{ه}}) + \text{ر}p(A_{\text{ه}}) + p(A_{\text{ه}}) = ١ \quad (\cdot / ٤\Delta)$ $p(A_{\text{ه}}) = \frac{١}{٩} \quad (\cdot / ٤\Delta) \qquad \qquad p(A_1) = \text{ر}p(A_{\text{ه}}) = \frac{\text{ر}}{٩} \quad (\cdot / ٤\Delta)$	ج
ج ذ ك	$p(a) = \text{ر}p(b) \quad (\cdot / ٣\Delta)$ $p(b) = \frac{١}{٣} p(c) \rightarrow \text{ر}p(b) = p(c) \quad (\cdot / ٣\Delta)$ $p(a) + p(b) + p(c) = ١ \quad (\cdot / ٣\Delta)$ $\text{ر}p(b) + p(b) + \text{ر}p(b) = ١ \quad (\cdot / ٣\Delta) \rightarrow p(b) = \frac{١}{٦} \quad (\cdot / ٣\Delta)$ $p(a) = \frac{\text{ر}}{٦} = \frac{١}{٦} \quad (\cdot / ٣\Delta) \qquad p(a') = ١ - \frac{١}{٦} = \frac{٥}{٦} = \frac{٥}{٦} \quad (\cdot / ٣\Delta)$	ج

	$P(a) = \tau p(b)$ $p(b) = \frac{1}{\lambda} p(c) \rightarrow p(c) = \tau p(b) \quad (\cdot / \lambda)$ $p(c) = p(d)$ $p(b) = w$ $p(a) + p(b) + p(c) + p(d) = 1 \quad (\cdot / 4\lambda) \quad \tau w + w + \tau w + \tau w = 1 \rightarrow w = \frac{1}{\lambda} \quad (\cdot / \lambda)$ $p(a) + p(d) = \tau w + \tau w = \lambda w = \frac{\lambda}{\lambda} \quad (\cdot / 4\lambda)$	✓
	$(\cdot / 4\lambda) \quad P(a) + P(b) + P(c) + P(d) = 1$ $(\cdot / 4\lambda) \quad P(a) + P(b) + P(c) = \frac{1\lambda}{4\lambda} \rightarrow P(d) = 1 - \frac{1\lambda}{4\lambda} = \frac{1\lambda}{4\lambda} \quad (\cdot / 4\lambda)$ $P(a) + P(d) = \frac{\lambda}{\lambda} \rightarrow P(a) = \frac{\lambda}{\lambda} - \frac{1\lambda}{4\lambda} = \frac{1}{4} \quad (\cdot / 4\lambda)$ $P(b) = P(c) \rightarrow \frac{\tau P(b)}{(\cdot / 4\lambda)} + \frac{\lambda}{\lambda} = 1 \rightarrow \tau P(b) = 1 - \frac{\lambda}{\lambda} = \frac{1}{4} \rightarrow P(b) = P(c) = \frac{1}{4}$	✓
	$p(\text{احمد}) + p(\text{علي}) + p(\text{بهرام}) = 1 \rightarrow \tau x + \tau x + x = 1 \rightarrow \tau x = 1 \rightarrow x = \frac{1}{\tau} \quad (\cdot / 4\lambda)$ $p(\text{علي}) = \tau p(\text{احمد}) = 2x$ $p(\text{بهرام}) = \tau p(\text{علي}) = \tau x$ $p(\text{احمد}) = x = \frac{1}{\tau} \quad (\cdot / 4\lambda) \quad p(\text{علي}) = \frac{1}{\tau} \quad (\cdot / 4\lambda) \quad p(\text{بهرام}) = \frac{1}{\tau} \quad (\cdot / 4\lambda)$	9
	$P(\gamma) = P(\tau) = P(\delta) = \tau x \quad (\cdot / 4\lambda) \Rightarrow \tau x + \tau x + \tau x + x + x + x = 1 \Rightarrow x = \frac{1}{12} \quad (\cdot / \delta)$ $P(1) = P(\tau) + P(\delta) = x \quad (\cdot / 4\lambda)$ $A = \{\gamma, \tau, \delta\} \rightarrow P(A) = x + \tau x + \tau x = \tau x = \frac{\tau}{12} \quad (\cdot / \delta)$	10
	$p\{a,b\} = \tau p\{c\} \Rightarrow p\{a\} + p\{b\} = \tau p\{c\} \quad (\cdot / \delta)$ $p\{a\} + p\{b\} + p\{c\} = 1 \quad (\cdot / \delta) \Rightarrow \tau p\{c\} + p\{c\} = 1 \Rightarrow p\{c\} = \frac{1}{\tau} \quad (\cdot / \delta)$	11

١٢

$$p(a) = r p(b) = rx$$

$$p(a) + p(b) + p(c) + p(d) = 1 \quad (\cdot / 4)$$

$$p(b) = x \quad (\cdot / 4)$$

$$rx + x + \frac{1}{4} + \frac{1}{4} = 1$$

$$p(c) = p(d) = \frac{1}{4}$$

$$rx = \frac{1}{4} \rightarrow x = \frac{1}{r} \quad (\cdot / 4)$$

$$p(a) = \frac{1}{r} \quad (\cdot / 4), \quad p(b) = \frac{1}{r}$$

$$p(a') = \frac{r}{4} \quad (\cdot / 4), \quad p(b') = \frac{1}{r} \quad (\cdot / 4)$$

الخطوة الأولى

١٣

$$P(a) = r P(b) = rx$$

$$p(a) + p(b) + p(c) = 1$$

$$p(b) = r p(c) = rx$$

$$rx + rx + x = 1$$

$$p(c) = x \quad (\cdot / 4)$$

$$x = \frac{1}{4} \quad (\cdot / 4)$$

$$\text{الـ } p(a) = \frac{r}{4} \quad (\cdot / 4)$$

$$\therefore P\{a, b\} = \frac{r}{4} + \frac{r}{4} = \frac{r}{2} \quad (\cdot / 4)$$

الخطوة الثانية

١٤

$$P(1) = P(2) = P(3) = rx \quad (\cdot / 3)$$

$$P(1) = P(2) = P(3) = x$$

$$P(1) + P(2) + \dots + P(3) = 1 \Rightarrow rx = 1 \Rightarrow x = \frac{1}{r} \quad (\cdot / 3)$$

$$A = \{1, 2, 3\} \Rightarrow P(A) = x + rx + x = rx = \frac{r}{r} \quad (\cdot / 3)$$

الخطوة الثالثة

الخطوة الرابعة

١٥

$$p(a) = \frac{1}{r} p(b) \quad , \quad p(a) + p(b) = 1 \quad \Rightarrow \frac{1}{r} p(b) + p(b) = 1 \quad \Rightarrow$$

$$p(b) = \frac{r}{r+1} \quad (\cdot / 4)$$

$$p(a) = \frac{1}{r+1} \quad (\cdot / 4)$$

$$A = r p(a) + \frac{1}{r} p(b) = \frac{r}{r+1} + \frac{1}{r+1} = \frac{1}{r+1} \quad (\cdot / 4)$$

١٦

$$P(1) = P(2) = P(3) = P(4)$$

$$\begin{aligned} p(4) &= x \rightarrow P(1) = 4x, \quad p(2) = 2x, \quad p(3) = \frac{4}{3}x \\ p(1)+p(2)+p(3)+p(4) &= 1 \Rightarrow 4x+2x+\frac{4}{3}x+x=1 \Rightarrow x=\frac{3}{25} \Rightarrow P(1)=\frac{12}{25} \end{aligned}$$

١٧

$$P(A) = P(B) \quad (\cdot / ٢٥)$$

$$P(C) = 2P(A) = 2P(B) \quad (\cdot / ٢٥)$$

$$P(A) + P(B) + P(C) = 1 \quad (\cdot / ٢٥)$$

$$P(A) + P(A) + 2P(A) = 1 \Rightarrow 4P(A) = 1 \Rightarrow P(A) = \frac{1}{4} \quad (\cdot / ٢٥) \quad \text{(الف)}$$

$$P(C) = 2P(A) = \frac{2}{4} = \frac{1}{2} \quad (\cdot / ٢٥)$$

$$P(\{A, C\}) = P(A) + P(C) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \quad (\cdot / ٢٥) \quad \text{(ب)}$$

٢٠

$$p(a) + p(b) + p(c) = 1 \quad (\cdot / ٢٥)$$

$$p(a) = \frac{1}{9} p(b)$$

$$p(b) = \frac{1}{3} p(c)$$

$$p(c) = x \quad \frac{1}{9}x + \frac{1}{3}x + x = 1 \Rightarrow x = \frac{9}{13} \quad (\cdot / ٢٥)$$

$$\text{(الف)} \quad p(a) = \frac{1}{9}, \quad p(b) = \frac{2}{9}, \quad p(c) = \frac{6}{9} \quad (\cdot / ٢٥) \quad (\cdot / ٢٥) \quad (\cdot / ٢٥)$$

$$\text{(ب)} \quad p\{b, c\} = p(b) + p(c) = \frac{2}{9} + \frac{6}{9} = \frac{8}{9} \quad (\cdot / ٢٥) \quad (\cdot / ٢٥)$$

٢٢

$$p(a) + p(b) + p(c) + p(d) = 1 \quad (\cdot / ٢٥)$$

$$2p(b) + p(b) + \frac{1}{4} + \frac{1}{8} = 1 \Rightarrow p(b) = \frac{3}{32} \quad p(a) = \frac{9}{32} \Rightarrow p(a') = 1 - \frac{9}{32} = \frac{23}{32} \quad (\cdot / ٢٥)$$

۲۳

$$\begin{aligned}
 P(C) &= x \\
 P(A) &= P(B) = rx \quad (\because \text{ر} \Delta) \\
 P(A) + P(B) + P(C) &= 1 \Rightarrow rx + rx + x = 1 \Rightarrow x = \frac{1}{\Delta} \Rightarrow P(A) = \frac{r}{\Delta}, P(B) = \frac{r}{\Delta}, P(C) = \frac{1}{\Delta} \\
 P(B \cup C) &= \frac{r}{\Delta} + \frac{1}{\Delta} = \frac{r+1}{\Delta} \quad (\because \Delta)
 \end{aligned}$$

۲۴

$$p(a) + p(b) + p(c) = 1 \quad (\because \text{ر} \Delta)$$

$$rp(b) + p(b) + \frac{1}{\text{ر}} = 1 \Rightarrow p(b) = \frac{1}{\text{ر}} \Rightarrow p(a) = \frac{1}{\text{ر}}$$

۲۵

$$p(\{b, c\}) = \frac{1}{\text{ر}} \Rightarrow p(b) + p(c) = \frac{1}{\text{ر}} \Rightarrow \frac{1}{\text{ر}} + p(c) = \frac{1}{\text{ر}} \Rightarrow p(c) = \frac{1}{\text{ر}} \quad (\because \text{ر} \Delta)$$

$$p(\{b, d\}) = \frac{1}{\text{ر}} \Rightarrow p(b) + p(d) = \frac{1}{\text{ر}} \Rightarrow \frac{1}{\text{ر}} + p(d) = \frac{1}{\text{ر}} \Rightarrow p(d) = \frac{1}{\text{ر}} \quad (\because \text{ر} \Delta)$$

$$p(a) + p(b) + p(c) + p(d) = 1 \Rightarrow p(a) = 1 - \left(\frac{1}{\text{ر}} + \frac{1}{\text{ر}} + \frac{1}{\text{ر}} \right) \Rightarrow p(a) = \frac{1}{\text{ر}} \quad (\because \text{ر} \Delta)$$

۲۶

$$\begin{aligned}
 p(\text{۱}) &= p(\text{۲}) = p(\text{۳}) = \text{ر}a \\
 p(\text{۴}) &= p(\text{۵}) = p(\text{۶}) = a
 \end{aligned} \quad (\because \Delta)$$

$$p(\text{۱}) + p(\text{۲}) + p(\text{۳}) + p(\text{۴}) + p(\text{۵}) + p(\text{۶}) = 1 \quad (\because \text{ر} \Delta)$$

$$\text{ر}a + a + \text{ر}a + a + \text{ر}a + a = 1$$

$$1\text{ر}a = 1 \Rightarrow a = \frac{1}{\text{ر}} \quad (\because \text{ر} \Delta)$$

$$p(A) = p(\text{۱}) + p(\text{۲}) = \frac{1}{\text{ر}} + \frac{\text{ر}}{\text{ر}} = \frac{\text{ر}}{\text{ر}} = 1 \quad (\because \text{ر} \Delta) = \frac{1}{\text{ر}}$$

۲۷	$S = \{a, b, c\}$ $p(a) = p(b) = p(c) \quad (\cdot/25)$ $p(a) + p(b) + p(c) = 1 \quad (\cdot/25)$ $p(c) + p(c) + p(c) = 1 \Rightarrow p(c) = \frac{1}{3} \quad (\cdot/25)$ $p(a) = p(b) = \frac{1}{3} \quad (\cdot/25)$ $p(\{b, c\}) = p(b) + p(c) \quad (\cdot/25) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \quad (\cdot/25)$
۲۸	$p(1) + p(2) + p(3) = 1 \xrightarrow{(\cdot/25)} a^2 + a/2 + a = 1$ $a^2 + 3a - 2 = 0 \quad (\cdot/25) \begin{cases} a = 1/2 \quad (\cdot/25) \rightarrow p(2) = 1/4 \quad (\cdot/25) \\ a = -2 \quad (\cdot/25) \end{cases} \quad (\cdot/25)$ غير قابل قبول (طبق اصل احتمال)
۲۹	$\begin{cases} P(1) = P(2) = P(3) = \omega \\ P(4) = P(5) = P(6) = \omega \\ A = \{2, 4, 6\} \end{cases} \quad (\cdot/25)$ $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1 \quad (\cdot/25) \Rightarrow 12\omega = 1$ $\Rightarrow \omega = \frac{1}{12} \quad (\cdot/25)$ $P(A) = P(2) + P(4) + P(6) \quad (\cdot/25) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{4} \quad (\cdot/25)$
۳۰	$P(1) = P(2) = P(3) = a \quad (\cdot/25)$ $P(4) = P(5) = P(6) = 2a \quad (\cdot/25)$ $a + 2a + 2a + a + 2a + a = 1 \Rightarrow a = \frac{1}{9} \quad (\cdot/25) \Rightarrow P(A) = \underbrace{P(2) + P(5) + P(6)}_{(\cdot/25)} = \frac{4}{9} \quad (\cdot/25)$
۳۱	احتمال در فضاهای پیوسته
۳۲	$a(s) = 2 \times 2 = 4 \quad (\cdot/25)$ $a(A) = a(\text{قطع} \ 2) = a(\text{دایره}) = \pi R^2 = \pi \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \quad (\cdot/25)$ $P(A) = \frac{a(A)}{a(s)} = \frac{\pi}{4} = \frac{\pi}{16} \quad (\cdot/25)$
۳۳	رسم شکل $(\cdot/25)$

$$S = [0, 1] \times [0, 1]$$

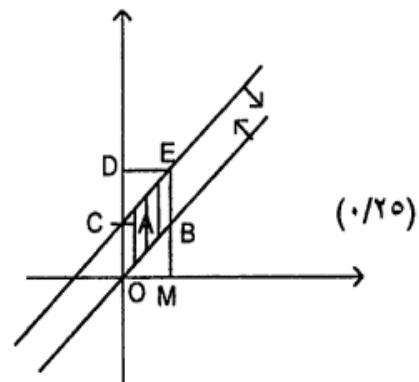
$$\begin{cases} x \leq y & (0,0) \\ y \leq x+1 & (0,1) \quad (-1,0) \end{cases} \quad (0/25)$$

$$a(s) = 1 \times 1 = 1 \quad (0/25)$$

$$a(A) = a(s) - [a(O M B) + a(C D E)] \quad (0/25)$$

$$a(A) = 1 - \left[\frac{1 \times 1}{2} + \frac{1 \times 1}{2} \right] \quad (0/25)$$

$$a(A) = 1 - 1 = 0 \quad (0/25) \quad P(A) = \frac{a(A)}{a(s)} = \frac{0}{1} = 0 \quad (0/25)$$

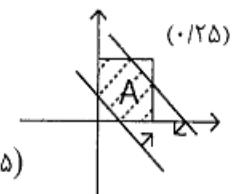


$$S = \{(x, y) \in R^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

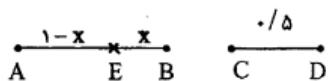
$$A = \{(x, y) \in R^2 \mid 0 \leq x+y \leq 1\}$$

$$a(s) = 1 \times 1 = 1 \quad (0/25) \quad \begin{cases} x+y \geq 0 \\ x+y \leq 1 \end{cases} \quad (0/25)$$

$$a(A) = 1 - \left(\frac{1 \times 1}{2} + \frac{1 \times 1}{2} \right) = 1 - \left(\frac{1}{2} + \frac{1}{2} \right) = 0 \quad (0/25) \quad P(A) = \frac{a(A)}{a(s)} = \frac{0}{1} = 0 \quad (0/25)$$



فرض می کنیم قطعه چوب ۱ متری در نقطه E برباد شود که به فاصله X از یک سر چوب قرار دارد. بنابراین فضای نمونه ای را می توان خط AB به طول ۱ متر در نظر گرفت (۰/۲۵)



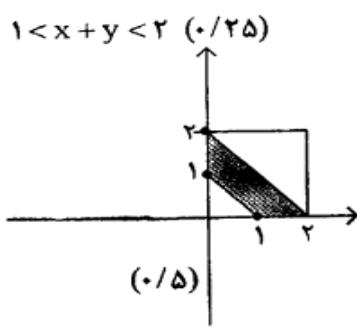
برای اینکه سه پاره خط AE, EB, CD، تشکیل یک مثلث بدهند باید طول هر پاره خط از مجموع طولهای دو پاره خط دیگر کمتر شود.

$$AE + EB > CD \Rightarrow 1 - x + x > \frac{1}{2} \Rightarrow 1 > \frac{1}{2}$$

$$AE + CD > EB \Rightarrow 1 - x + \frac{1}{2}x > \frac{1}{2} \Rightarrow \frac{1}{2}x > \frac{1}{2} \Rightarrow x < 1 \quad (0/25)$$

$$EB + CD > AE \Rightarrow x + \frac{1}{2} > 1 - x \Rightarrow 2x > \frac{1}{2} \Rightarrow x > \frac{1}{4}$$

$$A = \left\{ x \middle| \frac{1}{2} < x < \frac{3}{4} \right\} \quad (0/25) \quad P(A) = \frac{L(A)}{L(S)} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \quad (0/25)$$



$$a_s = 2 \times 2 = 4 \quad (\cdot / 2\Delta)$$

$$a_A = 4 - \left(\frac{2 \times 2}{2} + \frac{1 \times 1}{2} \right)$$

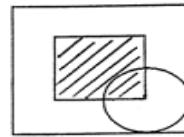
$$a_A = 4 - \frac{5}{2} = \frac{3}{2} \quad (\cdot / \Delta)$$

$$P(A) = \frac{a_A}{a_s} = \frac{\frac{3}{2}}{4} = \frac{3}{8} \quad (\cdot / \Delta)$$

$$a_s = a^2 = \Delta^2 = 2\Delta \quad (\cdot / 2\Delta)$$

$$a_A = a^2 = r^2 = 1 \quad (\cdot / 2\Delta)$$

$$P(A) = \frac{a_A}{a_s} = \frac{1}{2\Delta} \quad (\cdot / \Delta)$$



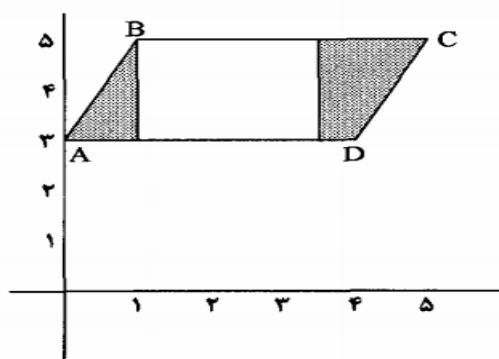
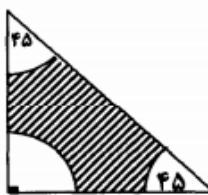
(· / 2Δ)

$$a_s = \frac{1}{2} \times 3 \times 3 = \frac{9}{2} \quad (\cdot / 2\Delta)$$

$$a_A = \frac{9}{2} - \frac{1}{2} \times \pi \times 1^2 = \frac{9 - \pi}{2} \quad (\cdot / 2\Delta)$$

$$P(A) = \frac{a_A}{a_s} = \frac{\frac{9 - \pi}{2}}{\frac{9}{2}} = \frac{9 - \pi}{9} \quad (\cdot / \Delta)$$

از مساحت مثلث مساحت نیم دایره کم می شود (· / 2Δ)

 $x > 3/\Delta$ یا $x < 1$

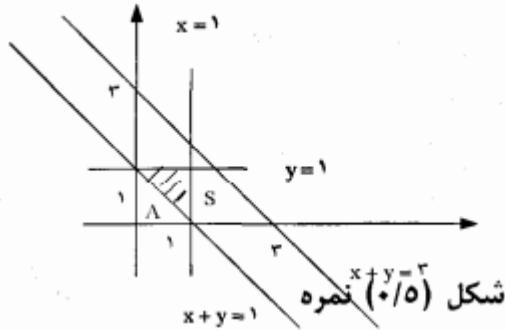
شکل (· / ۵)

$$a_s = 2 \times 2 = 4 \quad (\cdot / 2\Delta)$$

$$a_A = 4 - 2/\Delta \times 2 = 3 \quad (\cdot / 2\Delta) \quad \text{یا} \quad \left(a_A = \frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times 2(\cdot / \Delta + 1/\Delta) = 3 \right) \quad (\cdot / 2\Delta)$$

$$P(A) = \frac{a_A}{a_s} = \frac{3}{4} \quad (\cdot / 2\Delta)$$

۹۵



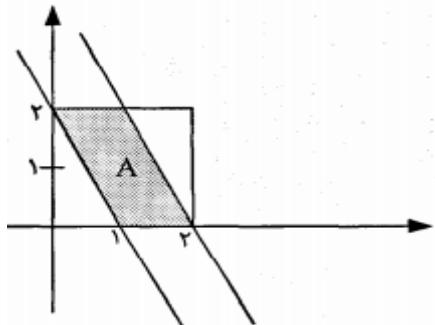
$$a(S) = \text{بی نهایت} \quad (\cdot / ۵)$$

$$a(A) = \frac{1 \times 1}{\gamma} = \frac{1}{\gamma} \quad (\cdot / ۲۵)$$

$$P(A) = \frac{a(A)}{a(S)} = \cdot \quad (\cdot / ۵)$$

۹۶

۹۶



$$y + \gamma x = \gamma$$

x	y
۱	۰
r	r

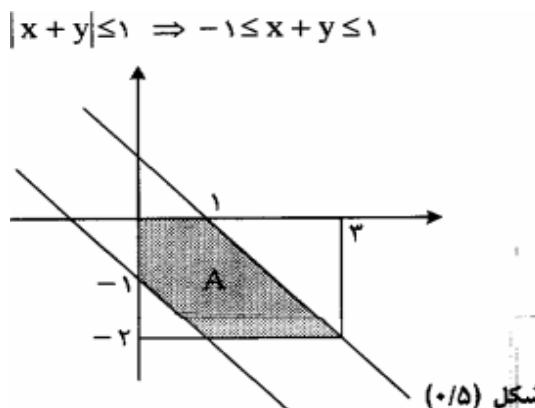
$$y + \gamma x = \gamma$$

x	y
۰	r
r	۰

$$P(A) = \frac{S_A}{S_S} = \frac{\gamma - (\frac{\gamma \times 1}{\gamma} + \frac{\gamma \times 1}{\gamma})}{\gamma} = \frac{1}{\gamma} \quad (\cdot / ۲۵)$$

۹۷

۹۷

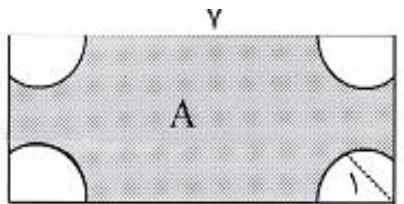


$$|x+y| \leq 1 \Rightarrow -1 \leq x+y \leq 1$$

$$P(A) = \frac{a_A}{a_S} = \frac{\gamma - (\frac{-1}{\gamma} + \gamma)}{\gamma} = \frac{\gamma}{\gamma} = \frac{\gamma}{\gamma} = \frac{1}{12} \quad (\cdot / ۵)$$

۹۸

۹۸

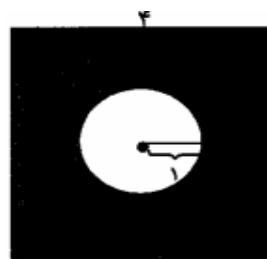


$$P(A) = \frac{a_A}{a_S} = \frac{\gamma r - \pi}{\gamma \times \gamma} = \frac{\gamma r - \pi}{\gamma \gamma} = 1 - \frac{\pi}{\gamma \gamma} \quad (\cdot / ۲۵)$$

(۹۸)

(۱ نمرہ)

جزداد ۸۸

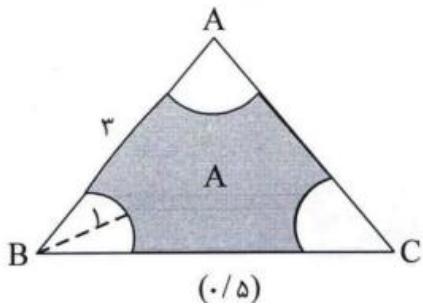


شکل (۸۸)

$$P(A) = \frac{a(A)}{a(S)} = \frac{\frac{1}{4}\pi - \pi}{\frac{1}{4}\pi} = \frac{-\frac{3}{4}\pi}{\frac{1}{4}\pi}$$

۱۳

جزداد ۸۹

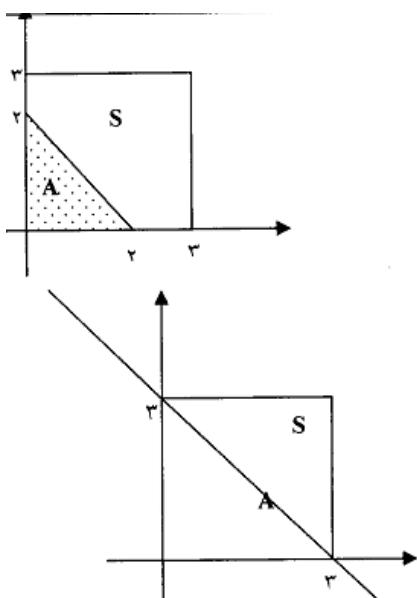


(۸۹)

$$P(A) = \frac{a(A)}{a(S)} = \frac{\frac{9\sqrt{3}}{4} - \frac{\pi}{2}}{\frac{9\sqrt{3}}{4}} = 1 - \frac{2\pi}{9\sqrt{3}}$$

۱۴

جزداد ۹۰



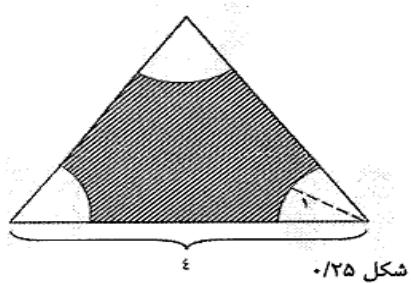
شکل (۹۰)

$$P(A) = \frac{\frac{2}{2} \times \frac{2}{2}}{9} = \frac{2}{9}$$

۱۵

(ب) چون خط مساحتی ندارد. (۰/۲۵) $P(B) = \frac{0}{9} = 0$

تکمیل پذیر ۰/۰

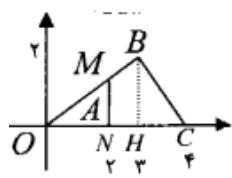


شکل (۹۱)

$$P(A) = \frac{a(A)}{a(S)} = \frac{\frac{\sqrt{3}}{4}a^2 - \pi}{\frac{\sqrt{3}}{4}a^2} = \frac{4\sqrt{3} - \pi}{4\sqrt{3}}$$

۱۷

٤٠



شكل (٤٠)

$$a(S) = \frac{1 \times 2}{2} = 1 \quad (\cdot / ٢٥)$$

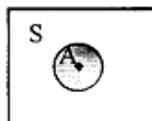
$$MN \parallel BH \Rightarrow \frac{MN}{BH} = \frac{ON}{OH} \Rightarrow \frac{MN}{2} = \frac{1}{2} \Rightarrow MN = 1 \quad (\cdot / ٢٥)$$

$$a(A) = \frac{ON \times MN}{2} = \frac{1 \times 1}{2} = \frac{1}{2} \quad (\cdot / ٢٥)$$

$$P(A) = \frac{a(A)}{a(S)} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \quad (\cdot / ٢٥)$$

٤٩

٤١



شكل (٤١)

(\cdot / ٢٥)

$$p(A) = \frac{a_A}{a_S} = \frac{\frac{1}{4} \pi}{1^2} = \frac{\pi}{4} \quad (\cdot / ٢٥)$$

(\cdot / ٢٥) (\cdot / ٢٥)

٤٠

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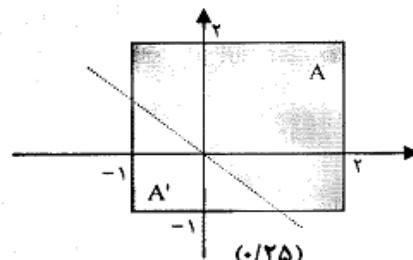
$$S = \{(x, y) \in R^2 \mid -1 \leq x \leq 2, -1 \leq y \leq 2\}$$

$$A = \{(x, y) \in S \mid x + y > 1\} \quad (\cdot / ٢٥)$$

$$a_S = 4 \quad (\cdot / ٢٥)$$

$$a_{A'} = \frac{2 \times 2}{2} = 2 \quad (\cdot / ٢٥) \Rightarrow a_A = 4 - 2 = 2 \quad (\cdot / ٢٥)$$

$$p(A) = \frac{a_A}{a_S} = \frac{2}{4} = \frac{1}{2} \quad (\cdot / ٢٥)$$



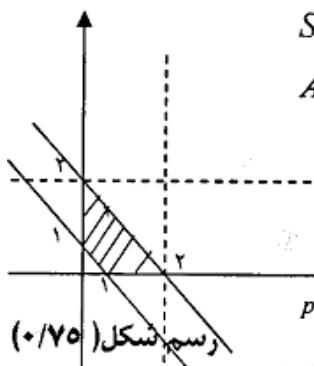
(\cdot / ٢٥)

٤٢

٤٣

$$S = \{(x, y) \mid 0 < x < 2, 0 < y < 2\} \quad (\cdot / ٢٥)$$

$$A = \{(x, y) \mid 1 < x + y < 2\} \quad (\cdot / ٢٥)$$



٤٣

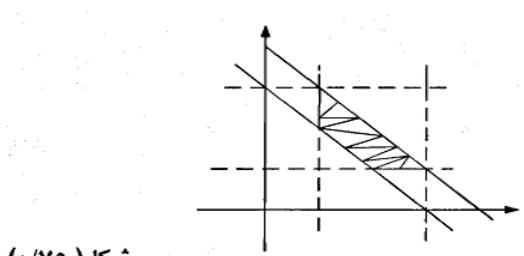
$$p(A) = \frac{a_A}{a_S} = \frac{2 - 1}{4} = \frac{1}{4} = \frac{1}{8} \quad (\cdot / ٢٥)$$

۲۴

$$S = \{(x, y) \mid 1 < x < 3, 1 < y < 3\} \quad (./25)$$

$$A = \{(x, y) \mid 3 < x + y < 4\} \quad (./25)$$

فرزند ۹۲



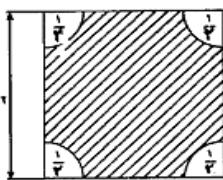
(+) ۷۵ رسم شکل

$$P(A) = \frac{a_A}{a_S} = \frac{4 - (\frac{1}{2} + 2)}{4} = \frac{\frac{3}{2}}{4} = \frac{3}{8} \quad (./25)$$

۲۵

در چهار گوشه مربع چهار ربع دایره به شعاع $\frac{1}{2}$ و به مرکز راس های مربع داریم که دایره ای به شعاع $\frac{1}{2}$ را تشکیل می دهند: (+/۵)

شناخت پرور ۹۲



$$a_S = 2 \times 2 = 4 \quad (./25)$$

$$a_A = 4 - \left(\frac{1}{2} \times \frac{1}{2} \times \pi \right) = 4 - \frac{\pi}{4} \quad (./25)$$

$$P(A) = \frac{a_A}{a_S} = \frac{4 - \frac{\pi}{4}}{4} = \frac{16 - \pi}{16} \quad (./5)$$

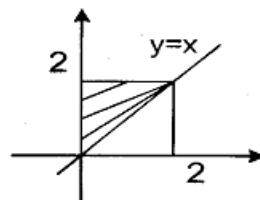
۲۶

۹۲

$$S = \{(x, y) \mid 0 < x < 2, 0 < y < 2\} \quad (+/25)$$

$$A = \{(x, y) \mid \frac{x}{Y} < 1\} \quad (./25)$$

$$P(A) = \frac{a_A}{a_S} = \frac{2}{4} = \frac{1}{2} \quad (./5)$$



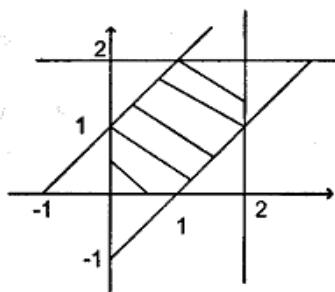
(+) ۷۵ رسم شکل

۲۷

$$S = \{(x, y) \mid 0 < x < 2, 0 < y < 2\} \quad (0/25)$$

$$A = \{(x, y) \mid -1 < x - y < 1\} \quad (0/25)$$

رسم شکل (۰/۵)



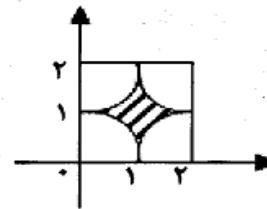
$$P(A) = \frac{a_A}{a_S} = \frac{\frac{4}{2} - (\frac{1}{2} + \frac{1}{2})}{\frac{4}{2}} = \frac{1}{2} \quad (0/25)$$

خرداد ۳۴

$$a_s = (2)^2 = 4 \quad (0/25)$$

$$a_A = 4 - \pi(1)^2 = 4 - \pi \quad (0/25)$$

$$P(A) = \frac{a_A}{a_S} \quad (0/25) = \frac{4 - \pi}{4} \quad (0/25)$$



رسم مربع در دستگاه مختصات مختصات (۰/۲۵)

ناحیهٔ سایه‌ده شده (۰/۲۵)

پژوهش

۹۳

۲۸

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq 2\}$$

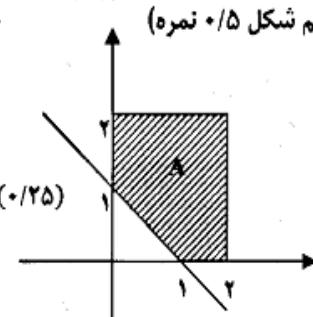
$$a_s = 2 \times 2 = 4 \quad \text{مساحت مربع} \quad (0/25)$$

$$A = \{(x, y) \in S \mid 1 \leq x + y\} \Rightarrow a_A = 4 - \text{مساحت مثلث سفید} - \text{مساحت مربع} = 4 - \frac{1}{2} = \frac{3}{2} \quad (0/25)$$

$$P(A) = \frac{a_A}{a_s} \quad (0/25) = \frac{\frac{3}{2}}{4} = \frac{3}{8} \quad (0/25)$$

ص ۱۰۹

(رسم شکل ۵/۰ نمره)

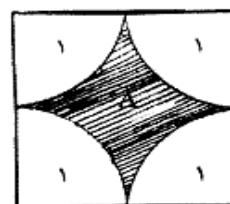


$$a_S = 2 \times 2 = 4 \quad (./25)$$

$$\text{مساحت دایره به شعاع ۱} = \pi \quad (./25)$$

$$a_A = 4 - \pi \quad (./25)$$

$$P(A) = \frac{a_A}{a_S} = \frac{4 - \pi}{4} \quad (./25)$$

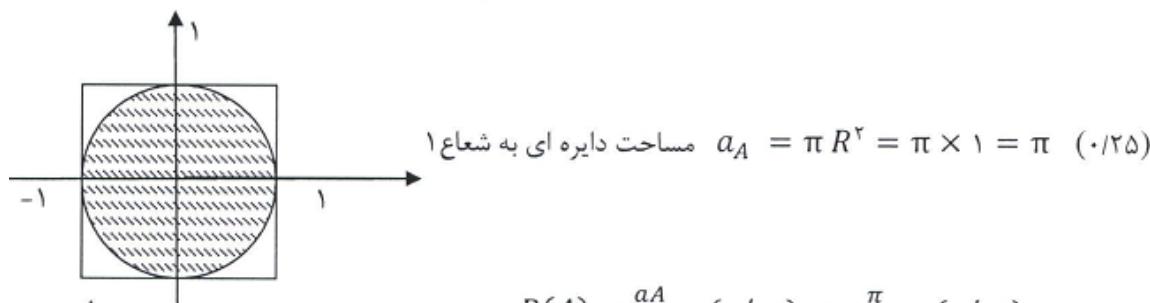


ص ۱۰۹

رسم شکل (./25)

۳۰

$$a_S = 2 \times 2 = 4 \quad (./25)$$

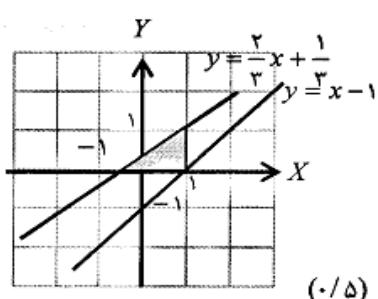


رسم فضای نمونه (./25)

رسم پیشامد (./25)

ص ۱۰۲ و ۱۰۳

۳۲



(./5)

$$a_S = 2 \times 1 = 2 \quad (./25), \quad a_A = \frac{1}{2} \times 1 \times \frac{3}{2} = \frac{3}{4} \quad (./25) \Rightarrow$$

$$P(A) = \frac{\frac{3}{4}}{2} = \frac{3}{8} \quad (./5)$$

صفحه ۱۰۷

۳۳

قوانين احتمال

$$P(A) = . / ۳۴ \quad P(B) = . / ۲۳ \quad P(A \cup B) = . / ۲۸ \quad (./25) \quad P(A \cap B) = ?$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (./25)$$

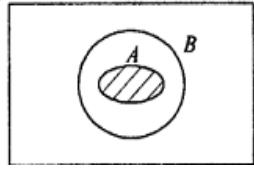
$$. / ۲۸ = . / ۳۴ + . / ۲۳ - P(A \cap B)$$

$$P(A \cap B) = . / ۳۴ + . / ۲۳ - . / ۲۸ \quad (./25)$$

$$P(A \cap B) = . / ۵۷ - . / ۲۸ \quad (./25)$$

$$P(A \cap B) = . / ۱۹ \quad (./25)$$

۱

۲۵ $A \cap (B - A) = \emptyset$ همچنین $B = (B - A) \cup A$ (۰/۲۵) $P(B) = P((B - A) \cup A)$ (۰/۲۵) $P(B) = P(B - A) + P(A)$ (۰/۲۵) $P(B - A) = P(B) - P(A)$ (۰/۲۵)	۲ 
۲۶ $P(B) = 1 - P(B') = 1 - 1/4 = 3/4$ (۰/۲۵) $1/4 = 1/3 + 1/6 - P(A \cap B)$ $P(A' \cap B') = 1 - P(A) - P(B) + P(A \cap B)$ (۰/۲۵) $P(A' \cap B') = 1 - 1/3 - 1/6 + 1/3 = 1/2$ (۰/۵)	۲ راه اول : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (۰/۲۵) $P(A \cap B) = 1/2$ (۰/۲۵) راه دوم : $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$ (۰/۲۵) $P(A' \cap B') = 1 - 1/2 = 1/2$ (۰/۲۵)
۲۷ $\frac{P(A)}{P(A')} = \frac{1}{3}$ (۰/۲۵) $\Rightarrow \frac{P(A)}{1 - P(A)} = \frac{1}{3}$ (۰/۲۵) \Rightarrow $3P(A) = 1 - P(A) \Rightarrow 4P(A) = 1 \quad P(A) = \frac{1}{4}$ (۰/۵)	۲
۲۸ <p style="text-align: center;">A: پیشامد بخش پذیر بودن عدد انتخابی بر ۲</p> $P(A) = \frac{\left[\frac{1 \dots}{2} \right]}{1 \dots} = \frac{5 \dots}{1 \dots}$ (۰/۲۵) <p style="text-align: center;">B: پیشامد بخش پذیر بودن عدد انتخابی بر ۷</p> $P(B) = \frac{\left[\frac{1 \dots}{7} \right]}{1 \dots} = \frac{142}{1 \dots}$ (۰/۲۵) $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{142}{1 \dots} = \frac{1}{1 \dots}$ (۰/۵) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5 \dots}{1 \dots} + \frac{142}{1 \dots} - \frac{1}{1 \dots} = \frac{571}{1 \dots}$ (۰/۵)	۲

	$P(A \cup B) \leq 1 \Rightarrow 1 - P(A \cup B) \geq 0 \quad (\cdot / 2\Delta)$ $1 - \left(P(A) + P(B) - P(A \cap B) \right) \geq 0 \quad (\cdot / \Delta)$ $\Rightarrow P(A \cap B) \geq P(A) + P(B) - 1 \quad (\cdot / 2\Delta)$	٦
	$p(A \cup B) = p(A) + p(B) - p(A \cap B) \quad (\cdot / 2\Delta)$ $p(A \cup B)' = 1 - p(A \cup B) = 1 - p(A) - p(B) + p(A \cap B) = 1 - p(A) - p(B) \quad (\cdot / 2\Delta)$	٧
	$p(A' \cap B') - p(A \cap B) = p(A \cup B)' - p(A \cap B) \quad (\cdot / \Delta)$ $= 1 - p(A \cup B) - p(A \cap B) \quad (\cdot / \Delta)$ $= 1 - p(A) - p(B) + p(A \cap B) - p(A \cap B) \quad (\cdot / 2\Delta)$ $= 1 - p(A) - p(B) \quad (\cdot / 2\Delta)$	٨٩
	$p(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (\cdot / 2\Delta)$ $P(B) = \cdot / 50 \quad (\cdot / 2\Delta)$ $P(A - B) = P(A) - P(A \cap B) = \cdot / 80 - \cdot / 4 = \cdot / 45 \quad \text{فقط يتحقق}$	٩
	$P(A) = \frac{\gamma}{\gamma} P(A \cap B)$ $P(B) = \frac{\gamma}{\gamma} P(A \cap B) \quad (\cdot / 2\Delta)$ $\frac{P(A \cup B)}{P(A \cap B)} = \frac{P(A) + P(B) - P(A \cap B)}{P(A \cap B)} = \frac{\frac{\gamma}{\gamma} P(A \cap B) + \frac{\gamma}{\gamma} P(A \cap B) - P(A \cap B)}{P(A \cap B)}$ $= \frac{\frac{\gamma}{\gamma} P(A \cap B)}{P(A \cap B)} = \frac{\gamma}{\gamma} \quad (\cdot / 2\Delta)$	١٠
	$(A \cap B') \cap (A \cap B) = A \cap (B' \cap B) = A \cap \emptyset = \emptyset \quad \text{از هم جدا هستند} \quad (\cdot / 2\Delta)$ $(A \cap B') \cup (A \cap B) = A \cap (B' \cup B) = A \cap U = A \quad (\cdot / 2\Delta)$ $P(A \cap B') + P(A \cap B) = P(A) \rightarrow P(A \cap B') = P(A) - P(A \cap B) \quad (\cdot / 2\Delta)$	١١

ج ن ج ن	$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) = \frac{1}{4} \Rightarrow P(A \cap B) = \frac{1}{4}$ $(./25)$ $P(A') = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$ $(./25)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \frac{1}{4} P(B) = \frac{3}{4} + P(B) - \frac{1}{4} \Rightarrow P(B) = \frac{1}{4}$ $(./25)$ $(./25)$	١٢
خ ر د د	$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) =$ $(./25) \quad (./25)$ $1 - [P(A) + P(B) - P(A \cap B)] = 1 - [P(A) + P(B) - P(A) \times P(B)] =$ $(./25) \quad (./25)$ $1 - [1/4 + 1/8 - 1/32] = 1 - 1/8 = 7/16 \quad (./25)$ $(./25)$	١٣
خ ر د د	$\text{أ) } (B - A) \cup (A \cap B) = (B \cap A') \cup (A \cap B) = B \cap (A \cup A') = B \cap U = B$ $(./25) \quad (./25) \quad (./25)$ $\text{ب) } (A \cap B \cap C)' = ((A \cap B) \cap C)' = (A \cap B)' \cup C' = A' \cup B' \cup C'$ $(./25) \quad (./25) \quad (./25)$	١٤
ج ن ج ن	$\begin{array}{l} : \text{مثلاً} \\ (B - A) \cup A = B \\ (B - A) \cap A = \emptyset \end{array} \Rightarrow P[(B - A) \cup A] = P(B) \quad (./25)$ $(./25)$ $P(B - A) + P(A) = P(B) \quad (./25) \Rightarrow P(B) \geq P(A) \quad (./25)$ $P(B - A) = P(B) - P(A) \quad (./25)$	١٥
خ ر د د	$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{5} \quad (./25)$ $(./5)$ $P(A \cap B') = P(A - B) = P(A) - P(A \cap B) = \frac{1}{5} \quad (./25)$ $(./5)$	١٦

۱۷	$P(A' \cap B') = p(A \cup B)' = 1 - p(A \cup B) =$ $(./\Delta) \quad (./\Delta)$ $1 - [p(A) + p(B) - p(A \cap B)] = 1 - p(A) - p(B) + p(A \cap B)$ $(./\Delta)$
۱۸	$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (./\Delta) \quad 1/75 = 1/55 + 1/60 - P(A \cap B)$ $P(A \cap B) = 1/45 \quad (./\Delta)$
۱۹	$A = 3 \quad \text{بخش پذیری بر } 3 \Rightarrow P(A) = \frac{[1...]}{1...} = \frac{333}{1...}$ $B = 5 \quad \text{بخش پذیری بر } 5 \Rightarrow P(B) = \frac{[1...]}{1...} = \frac{4...}{1...} \quad (./25)$ $P(A \cap B) = \frac{[1...]}{1...} = \frac{66}{1...} \quad (./\Delta)$ $P(A' \cap B) = P(B) - P(A \cap B) = \frac{4...}{1...} - \frac{66}{1...} = \frac{134}{1...} \quad (./25)$
۲۰	$\text{تشخیص پیدا کردن اشتراک دو پیشامد } 25/0 \text{ نمره دارد.}$ $P(A \cap B) = P(A) + P(B) - P(A \cup B) \Rightarrow P(A \cap B) = 1/25 + 1/30 - 1/4 = 1/12 \quad (./25)$
۲۱	$(d) \text{ درست}$
۲۲	$P(A \cup B) = p(A) + p(B) - p(A \cap B) \quad (./\Delta)$ $\frac{2}{3} = p(A) - p(A \cap B) + \frac{1}{4} \quad (./\Delta)$ $\frac{2}{3} = p(A - B) + \frac{1}{4} \Rightarrow p(A - B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12} \quad (./\Delta)$

٢٦

عدد مضرب ٣ باشد:

$$n(s) = 1000 \quad (./25) \quad n(A) = \left[\frac{1000}{3} \right] = 333 \quad (./25)$$

عدد مضرب ٥ باشد:

$$n(B) = \left[\frac{1000}{5} \right] = 200 \quad (./25) \quad n(A \cap B) = \left[\frac{1000}{15} \right] = 66 \quad (./25)$$

(الف)

$$P(A \cup B) = p(A) + p(B) - p(A \cap B) \quad (./25)$$

$$p(A \cup B) = \frac{333}{1000} + \frac{200}{1000} - \frac{66}{1000} = \frac{467}{1000} \quad (./25)$$

$$p(A - B) = p(A) - p(A \cap B) \quad (./25)$$

$$p(A - B) = \frac{333}{1000} - \frac{66}{1000} = \frac{267}{1000} \quad (./25)$$

(ب)

٢٧

اگر A پیشامد بخش پذیر بودن عدد انتخابی بر ٣ و B پیشامد بخش پذیر بودن عدد انتخابی بر ٥ باشد آنگاه:

$$\left. \begin{array}{l} P(A) = \frac{333}{1000} \quad (./5) \\ P(B) = \frac{200}{1000} \quad (./5) \\ P(A \cap B) = \frac{66}{1000} \quad (./5) \end{array} \right\} \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (./25)$$

$$P(A \cup B) = \frac{333}{1000} + \frac{200}{1000} - \frac{66}{1000} = \frac{467}{1000} \quad (./25)$$

٢٨

$$n(A \cap B) = \left[\frac{1000}{18} \right] = 55 \quad (./25) \quad n(A) = \left[\frac{1000}{4} \right] = 250 \quad (./25)$$

$$P(A \cap B') = p(A) - p(A \cap B) \quad (./25) \Rightarrow P(A \cap B') = \frac{250}{1000} - \frac{55}{1000} = \frac{195}{1000} \quad (./25)$$

٢٩

(الف) $P(B) = 1 - p(B') \quad (./25) = 1 - \frac{3}{4} = \frac{1}{4} \quad (./25)$

$$P(A \cup B) = p(A) + p(B) - p(A \cap B) \quad (./25) = \frac{2}{5} + \frac{1}{4} - \frac{1}{5} = \frac{27}{25} \quad (./25)$$

$$(ب) p(A - B) = P(A) - P(A \cap B) \quad (./25) = \frac{2}{5} - \frac{1}{5} \quad (./25) = \frac{1}{5}$$

۳۱	<p>پیش‌بینی</p> <p>هر مورد داشته باشد: $p(A) = \frac{1}{5}$, $p(B) = \frac{1}{5}$, $p(A \cup B) = \frac{1}{3}$ (۰/۲۵)</p> $p(A \cap B) = p(A) + p(B) - p(A \cup B) \quad (۰/۰) = \frac{1}{5} + \frac{1}{5} - \frac{1}{3} = \frac{1}{15} \quad (۰/۰)$	۳۱
۳۲	<p>گردش</p> <p>الف: $p(A') = \frac{1}{5}$ $p(A) = 1 - p(A') \quad (۰/۲۵) = 1 - \frac{1}{5} = \frac{4}{5} \quad (۰/۲۵)$</p> $p(A \cup B) = p(A) + p(B) - p(A \cap B) \quad (۰/۰) = \frac{4}{5} + \frac{1}{3} - \frac{1}{15} = \frac{13}{15} \quad (۰/۰)$ <p>ب): $p(A - B) = p(A) - p(A \cap B) \quad (۰/۰) = \frac{4}{5} - \frac{1}{15} = \frac{1}{3} \quad (۰/۰)$</p>	۳۲
۳۳	<p>پیش‌بینی</p> <p>$P(A \cap B) \leq 1 \quad (۰/۰)$: می‌دانیم</p> <p>$P(A \cup B) \leq 1 \quad (۰/۰) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1 \quad (۰/۰)$: می‌دانیم</p> $\Rightarrow 1 + 1 - P(A \cap B) \leq 1 \Rightarrow P(A \cap B) \geq 1 \quad (۰/۰)$ <p>$P(A \cap B) \leq 1$ و $P(A \cap B) \geq 1 \Rightarrow P(A \cap B) = 1 \quad (۰/۰)$ ص ۱۲۱</p>	۳۳
۳۴	<p>صفحه ۱۲۱</p> <p>A و B : مرد بودن و فوق لیسانس داشتن</p> $P(A) = \frac{۳۲}{۵۰} \quad (۰/۰), \quad P(B) = \frac{۲۰}{۵۰} \quad (۰/۰), \quad P(A \cap B) = \frac{۱۱}{۵۰} \quad (۰/۰)$ $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - \underbrace{(P(A) + P(B) - P(A \cap B))}_{(۰/۰)} = 1 - \frac{۴۱}{۵۰} = \frac{۹}{۵۰} \quad (۰/۰)$	۳۴

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